Extra Dimensions and Higgs Pair Production at Photon Colliders

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Abstract

We show that new physics effects due to extra dimensions can dramatically affect Higgs pair production at photon colliders. We find that the cross section due to extra dimensions with the scale M_S of new physics around 1.5 TeV, the cross section can be as large as 0.11 pb (1.5pb) for monochromatic photon collision, $\gamma\gamma \to HH$, with the collider energy $\sqrt{s} = 0.5(1)$ TeV for Higgs mass of 100 (350) GeV. The cross section can be 3 fb (2.7 fb) for the same parameters for collisions using photon beams from electron or positron back scattered by laser. These cross sections are much larger than those predicted in the Standard Model. Higgs pair production at photon colliders can provide useful tests for new physics due to extra dimensions.

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It has recently been proposed that gravitational effects can become large at a scale M_S near the weak scale due to effects from extra dimensions [1,2], quite different from the traditional thought that gravitational effects only become large at the Planck scale $M_{Pl} \sim 10^{19}$ GeV. In this proposal the total space-time is $D = 4 + n$ dimensions. When the extra dimensions are compactified there are towers of states, the Kaluza-Klein (KK) states, interact with ordinary matter fields. Although the interactions with the Standard Model (SM) fields for each of the KK state is small, proportional to the Newton constant G_N, the effects become much stronger, proportional to $1/M_S^2$, when the contributions of all the KK states are summed over.

The relation between the scale M_S and the Planck scale M_{Pl}, assuming all extra dimensions are compactified with the same size R, is given by, $M_{Pl}^2 \sim R^n M_S^{2+n}$. With M_S near a TeV, for n= 1 R would be too large which is ruled out. However, with n larger than or equal to 2, the theory is not ruled out. In these cases, R can be in the sub-millimeter region which can be probed by laboratory experiments [3–8]. The lower bound for M_S is constrained, typically, to be of order one TeV from present experimental data [3–8]. Future experiments will provide more stringent constraints. It is important to investigate as many systems as possible to give constraints on the scale M_S and to look for deviations from predictions in the to isolate effects due to extra dimensions. In this paper we study effects due to extra dimensions on Higgs pair production at photon colliders. We find that the cross section for this process can be dramatically different from SM predictions.

In the minimal Standard Model there is a neutral Higgs boson H resulting from spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ due to the Higgs mechanism. The mechanism for spontaneous symmetry breaking is not well understood. There is no experimental evidence favoring any particular mechanism, such as the Higgs mechanism. The discovery of the Higgs boson and understanding of its properties are fundamentally important [9]. The Higgs boson is also the last particle yet to be discovered in the minimal SM. The discovery of Higgs boson is one of the most important goals for future high energy colliders. At present the lower bound on the SM Higgs boson mass m_h is set by LEP II to be

95.5 GeV at 95% C.L. [10]. Many methods have been proposed to produce and to study the properties of Higgs bosons [9]. Higgs pair production at various colliders are some of the interesting ones [11–13]. In particular Higgs pair production through $\gamma\gamma \to HH$ is important for the study of Higgs boson properties. In SM the process $\gamma\gamma \to HH$ only occur at loop level with cross section of order O(0.5 fb) for $\sqrt{s} \sim (500 \ \text{to} \ 1000)$ and m_h in the range 100 to 400 GeV [12]. It is in principle reachable [14], but a formidable challenge for accelerator physics. The smallness of cross section predicted in SM makes the process, $\gamma\gamma \to HH$, sensitive to new physics beyond the minimal SM. New physics beyond SM may dramatically change the situation and increase the cross section to a level more practical for experiments [13]. In the following we show that this is indeed possible with extra dimensions.

After compactifying the extra n dimensions, for a given KK level $\vec{l}$ there are one spin-2 state, n-1 spin-1 and n(n-1)/2 spin-0 states [7]. Assuming that all SM fields are confined to a four dimensional world-volume and gravitation is minimally coupled to SM fields, it was found that the spin-1 KK states decouple while the spin-2 and spin-0 KK states couple to all SM fields [7]. Since the KK states interact with particles in the SM, exchanges of KK states can generate new interactions among the SM particles. For $\gamma\gamma \to HH$ we find that contrary to the situation in SM, this process can happen at the tree level due to exchange of spin-2 KK states as shown in Fig.1. Spin-1 and spin-0 KK states do not contribute to this process. Using the Feynmann rules given in Ref. [7], we have

$$\begin{aligned}
\widetilde{M}(\gamma\gamma \to HH) &= \frac{\kappa^2}{8}(m_h^2\eta^{\mu\nu} - C^{\mu\nu,\rho\sigma}k_{1\rho}k_{2\sigma}) \\
&\times \frac{B_{\mu\nu,\alpha\beta}}{q^2 - m_l^2}(p_1 \cdot p_2 C^{\alpha\beta,\delta\gamma} + D^{\alpha\beta,\delta\gamma})\epsilon_{1\delta}(p_1)\epsilon_{2\gamma}(p_2), \\
C^{\mu\nu,\rho\sigma} &= \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}, \\
D^{\alpha\beta,\delta\gamma} &= \eta^{\alpha\beta}k_1^\delta k_2^\gamma - [\eta^{\alpha\gamma}k_1^\beta k_2^\delta + \eta^{\alpha\delta}k_1^\gamma k_2^\beta - \eta^{\delta\gamma}k_1^\alpha k_2^\beta + (\alpha \to \beta, \beta \to \alpha)], \\
B_{\mu\nu,\alpha\beta} &= (\eta_{\mu\alpha} - \frac{q_\mu q_\alpha}{m_l^2})(\eta_{\nu\beta} - \frac{q_\nu q_\beta}{m_l^2}) + (\eta_{\nu\alpha} - \frac{q_\nu q_\alpha}{m_l^2})(\eta_{\mu\beta} - \frac{q_\mu q_\beta}{m_l^2}) \\
&- \frac{2}{3}(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m_l^2})(\eta_{\alpha\beta} - \frac{q_\alpha q_\beta}{m_l^2}). \qquad (1)
\end{aligned}$$

where $\kappa^2 = 16\pi G_N$, and m_l is KK state mass. After a straight forward calculation and

summing over all KK state contributions, we have

$$
\begin{aligned}
M = & -i\frac{\kappa^2 D_n(s)}{4}[2u\epsilon_1\cdot k_1\epsilon_2\cdot k_2 + 2t\epsilon_1\cdot k_2\epsilon_2\cdot k_1 \\
& + \epsilon_1\cdot\epsilon_2(tu - m_h^4) + 2m_h^2(\epsilon_1\cdot k_1\epsilon_2\cdot k_1 + \epsilon_1\cdot k_2\epsilon_2\cdot k_2)],
\end{aligned}
\tag{2}
$$

where $s = q^2$, $t = (k_1-p_1)^2 = (p_2-k_2)^2$, $u = (k_1-p_2)^2 = (k_2-p_1)^2$, $D_n(s) = \sum_l i/(s-m_l^2)$. With the cut-off scale for the summation set to be the same as M_S, one has

$$
\begin{aligned}
\kappa^2 D_n(s) &= \frac{16\pi}{M_S^4}\left(\frac{s}{M_S^2}\right)^{n/2-1}[\pi + i2I_n(M_S/\sqrt{s})], \\
I_n(x) &= \begin{cases} -\sum_{k=1}^{n/2-1}\frac{1}{2k}x^{2k} - \frac{1}{2}\ln(x^2-1), & \text{for } n = \text{even}, \\ -\sum_{k=1}^{(n-1)/2}\frac{1}{2k-1}x^{2k-1} + \frac{1}{2}\ln\frac{x+1}{x-1}, & \text{for } n = \text{odd}. \end{cases}
\end{aligned}
\tag{3}
$$

For $M_S^2 \gg s$, $\kappa^2 D_n(s)$ is approximately equal to $i(16\pi/M_S^4)\ln(M_S^2/s)$ for $n = 2$ and $i(16\pi/M_S^4)2/(n-2)$ for $n > 2$. There are a few different definitions of M_S used in the literature, although the same order of magnitude there are some differences numerically. We will use M_S defined above for our discussions [8].

The helicity amplitudes for different photon polarizations are given by

$$
\begin{aligned}
& M((1,+),(2,+)) = M((1,-),(2,-)) = \frac{\kappa^2 D_n(s)}{16}s^2(1-\frac{4m_h^2}{s})\sin^2\theta, \\
& M((1,+),(2,-)) = M((1,-),(1,+)) = 0,
\end{aligned}
\tag{4}
$$

where θ is the angle between the photon beam line and the out going Higgs boson momentum. $(1,\pm)$ and $(2,\mp)$ indicate the photon polarization vectors with $\epsilon_{1,\mu}(\pm) = \epsilon_{2,\mu}(\mp) = (0,\mp 1, 0, -i)/\sqrt{2}$. The cross section for polarized photons is given by,

$$
\begin{aligned}
& \frac{d\sigma((1,\pm),(2,\pm))}{d\cos\theta} = \frac{\pi}{64M_S^2}\left(\frac{s}{M_S^2}\right)^{n+1}[\pi^2 + 4I_n^2(M_S/\sqrt{s})]\sin^4\theta\left(1-\frac{4m_h^2}{s}\right)^{5/2}, \\
& \frac{d\sigma((1,\pm),(2,\mp))}{d\cos\theta} = 0.
\end{aligned}
\tag{5}
$$

The cross section for the unpolarized photon beam, $d\sigma(s)/d\cos\theta$ is the same as the above. We will use this case for later discussions.

In SM the lowest order contributions to $\gamma\gamma \to HH$ occur at one loop level. The cross section is predicted to be small. For monochromatic photon beam with $\sqrt{s} = 0.5(1)$ TeV

and the Higgs mass m_h in the range of $100 \sim 200(100 \sim 400)$ GeV, the cross section $\sigma(s)$ is in the range $0.8 \sim 0.4(0.6 \sim 0.5)$ fb [12]. These cross sections are small and difficult to achieve experimentally although not impossible. With effects from extra dimensions the cross section can be dramatically different.

The effects of extra dimensions on the Higgs pair production at photon colliders depend on the scale M_S, the number of extra dimensions n, the Higgs boson mass m_h and the collider energy $\sqrt{s}$. The cross section increases rapidly with s which is different than the prediction of SM where the cross section does not change dramatically with $\sqrt{s}$. The cross section decreases when M_S, m_h and n are increases. In our numerical analyses, we will take two typical value 1.0 and 1.5 TeV for the scale M_S and vary n up to 7 and m_h in typical kinematic allowed ranges for illustration [1].

The results for $\sigma(s)$ with monochromatic photon beams are shown in Table 1. From Table 1, we see that for $\sqrt{s} = 0.5$ TeV and m_h as high as 200 GeV, the cross sections can be as large as 14 fb and 274 fb for $M_S = 1.5$ TeV and 1 TeV, respectively. Even with n=7, the cross section can still reach 11 fb for $M_S = 1$ TeV. For $\sqrt{s} = 1$ TeV, the cross sections are even larger. For the worst case of n =7, for m_h as large as 450 GeV the cross section can still reach 12 fb and 3.9 pb with $M_S = 1.5$ TeV and 1.01 TeV, respectively. We also analyzed the case with $\sqrt{s} = 250$ GeV. In this case, the cross section for $M_S = 1.5$ TeV the cross section is small (< 0.34 fb) with $M_S = 1.5$ TeV. But the cross section can still be as large as 6.6 fb for $m_h = 100$ GeV with $M_S = 1.01$ TeV. It is clear that the cross sections for Higgs pair production at photon colliders with extra dimensions can be much larger than the SM predictions.

High energy monochromatic photon beams may be difficult to obtain. A practical method

[1]There is a singularity when $\sqrt{s} = M_S$ due to our negligence of the decays of the KK states. To avoid this artificial singularity, for $\sqrt{s} = 1$ TeV we use $M_S = 1.01$ TeV such that M_S is sufficiently away from $\sqrt{s}$.

to obtain high energy photon beams is to use laser back-scattering technique on an electron or positron beam which produces abundant numbers of hard photons nearly along the same direction as the original electron or positron beam. The photon beam energy obtained this way is not monochromatic. The energy spectrum of the back-scattered photon is given by [14]

$$F(x) = \frac{1}{D(\xi)}[1 - x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2}],$$
$$D(\xi) = (1 - \frac{4}{\xi} - \frac{8}{\xi^2}) \ln(1+\xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1+\xi)^2}, \tag{6}$$

where x is the fraction of the energy of the incident electron. The parameter ξ is determined to be $2(1+\sqrt{2})$ by requiring that the back-scattered photon to have the largest possible energy, but does not interfere with the incident photon to create unwanted e^+e^-. The maximal value x_{max} can be reach by x is given by $\xi/(1+\xi) \approx 0.828$.

The cross section $\sigma(e^+e^-)$ due to the back-scattered photon beams collide to produce Higgs pair, $e^+e^- \to \gamma\gamma \to HH$, is obtained by folding in the photon luminosities with the cross section $\sigma(s)$. We have

$$\sigma(e^+e^-) = \int_{x_{1min}}^{x_{max}} \int_{x_{2min}}^{x_{max}} F(x_1)F(x_2)\sigma(x_1x_2s)\theta(1 - \frac{4m_h^2}{x_1x_2s})dx_1dx_2, \tag{7}$$

with $x_{1min,2min} = 4m_h^2/(x_{max}s)$. Here $\sqrt{s}$ is the total energy of the $e^\pm$ beams.

The results for the same cases discussed for monochromatic photon beams are shown in Table 2. The cross section $\sigma(e^+e^-)$ is smaller than $\sigma(s)$ due to the replacement of s to x_1x_2s. However, we still find that the cross section can be much larger than that predicted by SM. In SM for $\sqrt{s} = 0.5$TeV and 1 TeV become smaller (< 0.2 fb) compared with the case for monochromatic photon beam. With effects from extra dimensions, for $\sqrt{s} = 0.5$ TeV, the cross section $\sigma(e^+e^-)$ can still be as large as 59 (3) fb for $m_h = 100$ GeV and 9.4(0.48) fb for $m_h = 150$ GeV with $M_S = 1.0(1.5)$ TeV. For $\sqrt{s} = 1$ TeV, the cross sections are much larger. $\sigma(e^+e^-)$ can be as large as 60 (2.7) fb for m_h as large as 350 GeV with $M_S = 1.01(1.5)$ TeV. For smaller m_h, the cross sections are even larger. These cross sections are much larger than SM predictions. Observations of Higgs pair events at photon colliders

at a level larger than 1 fb would be indications of effects from extra dimensions. If the effects due to extra dimensions indeed exist, production of Higgs pair at photon colliders may even become practically possible. Pair production of Higgs bosons at photon colliders can provide important information about effects from extra dimensions.

We remark that the angular distribution can also provide important information about the mechanism for Higgs pair production at photon colliders. For monochromatic photon colliders the angular distribution is simply given by $d\sigma/(\sigma d\cos\theta) = 15\sin^4\theta/16$ which peaks at $\theta = \pi/2$. For back scattered photon beams, the angular distribution $d\sigma(e^+e^-)/(\sigma(e^+e^-)d\cos\theta)$ is more complicated. Here θ is the angle between the $e^\pm$ beam direction and the momentum for one of the Higgs boson in the final state. The distribution also peaks at $\theta = \pi/2$ which is different than effects due to SM and other new physics, such anomalous triple Higgs boson coupling [12]. Also the formula obtained in Eq. (5) can be used for $gg \to HH$ with a simple replacement of photons to gluons. Effects of extra dimensions on Higgs pair production can also be studied at hadron colliders.

To conclude, we have shown that effects due to extra dimensions can induce tree level $\gamma\gamma \to HH$ scattering. Within the allowed range for the scale M_S the predicted cross sections can be much larger than those predicted by the Standard Model where $\gamma\gamma \to HH$ only occur at loop level. New physics due to extra dimensions can be tested using Higgs pair production at photon colliders. If the effects due to extra dimensions exist, production of Higgs pairs at photon colliders may even become practically possible.

Note added. After we have finished this work, we became to aware the work by Rizzo in Ref. [15] where among other things similar calculation was also done for $\gamma\gamma \to HH$. Our formula for this process agree.

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TABLES

TABLE I. The cross section $\sigma(s)$ for $\gamma\gamma \to HH$. a) $\sigma(s)$ (in unit fb) for $\sqrt{s} = 0.5$ TeV with $M_S = 1.00$ (1.50) TeV. b) $\sigma(s)$ (in unit pb) for $\sqrt{s} = 1.0$ TeV with $M_S = 1.01$ (1.5) TeV.

m_h (GeV)	n=2	n=3	n=4	n=5	n=6	n=7
a)(fb)						
100	2280(114)	941(34)	461(13)	249(6.09)	145(3.273)	91(1.98)
150	1155(58)	477(17)	233(6.66)	127(3.09)	74(1.66)	46(1.00)
200	274(14)	113(4.08)	55(1.58)	30(0.73)	18(0.39)	11(0.24)
b)(pb)						
100	427(7.12)	344(3.76)	297(2.27)	266(1.46)	243(9.78)	225(6.78)
200	306(5.10)	246(2.70)	213(1.62)	190(1.04)	174(7.00)	161(4.85)
350	88(1.46)	71(0.77)	61(0.47)	55(0.30)	50(0.20)	46(0.14)
450	7.4(0.124)	6.0(0.066)	5.2(0.039)	4.6(0.025)	4.2(0.017)	3.9(0.012)

TABLE II. The cross section $\sigma(e^+e^-)$ for $e^+e^- \to \gamma\gamma \to HH$. a) $\sigma(e^+e^-)$ (in unit fb) for $\sqrt{s} = 0.5$ TeV with $M_S = 1.00$ (1.50) TeV. b) $\sigma(e^+e^-)$ (in unit fb) for $\sqrt{s} = 1.0$ TeV with $M_S = 1.01$ (1.5) TeV.

m_h (GeV)	n=2	n=3	n=4	n=5	n=6	n=7
a)(fb)						
100	59(3.03)	19(0.71)	7.6(0.24)	3.6(0.10)	1.98(0.06)	1.20(0.03)
150	9.4(0.48)	3.1(0.12)	1.3(0.04)	0.63(0.02)	0.34(0.009)	0.21(0.006)
b)(fb)						
100	5615(271)	3094(106)	1954(50)	1322(27)	936(15)	683(9.5)
200	2607(124)	1481(50)	956(24)	657(13)	470(7.5)	347(4.7)
350	60(2.72)	37(1.17)	25(0.59)	18(0.33)	14(0.20)	10(0.12)

FIGURES

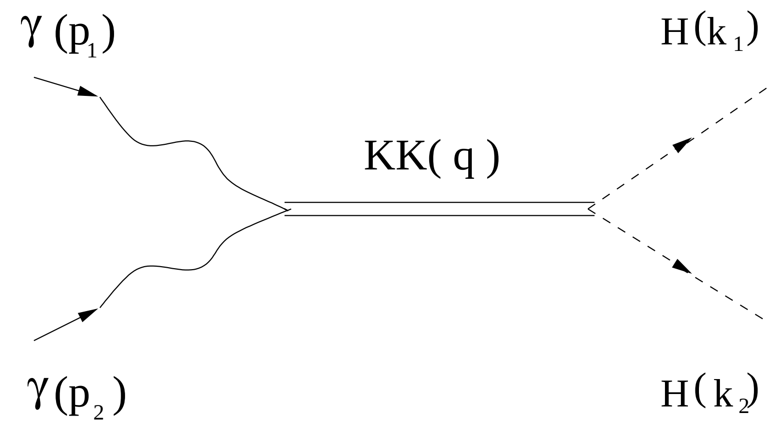

FIG. 1. Tree diagram for KK states contribution to $\gamma(p_1)\gamma(p_2) \to H(k_1)H(k_2)$.